LEARNING SEMINAR ON PRISMATIC COHOMOLOGY

1. Overview

Prismatic cohomology is a cohomology theory for algebraic varieties over p-adically complete rings that unifies various (integral) cohomology theories in the sense that they can be recovered as speical cases. In this seminar, we aim for introducing the notions of prismatic cohomology and its applications.

We will mainly follow Eilenberg lecture note by Bhatt [1] and *Prism and prismatic Cohomology* by Bhatt and Scholze [2]. The other useful references include [3].

2. Talks

2.1. **Overview talk.** Introductory talk.

2.2. **Delta rings.** The speaker will discuss the definition and examples of δ-rings , p-derivations and Frobenius lifts, the category of δ-rings and its properties (adjunction, free objects). The speaker should explain how Witt vectors induces an equivalence between the category of perfect rings of characteristic p and the category of p-adically complete perfect δ-rings.

2.3. **Distinguised elements and prisms.** The speaker will discuss the definition, example and properties of distinguished elements and prisms. The speaker should discuss on derived completions. It is necessary to have a good theory of completions along an ideal to work effectively with prisms. Unfortunately, the rings (resp. modules) that we shall encounter are often non-Noetherian (resp. not finitely generated). In this setting, the classical theory of completion does not behave so well. This defect is remedied by the theory of derived completions. The speaker should explain how to define derived completions and its properties.

2.4. **Perfect prisms and perfectoid rings.** The speaker will discuss the notion of a perfect prism. Here we will pause briefly to explain the notion of a perfectoid ring. All of this will be done with a view towards establishing an equivalence of categories between the category of perfect prisms, and the category of perfectoid rings. To go from perfectoid rings to perfect prisms, we will need Witt vectors. So, here we will take a minute to go through the basics of Witt vectors (e.g. construction, some basic examples to keep in mind, and basic properties that will be useful). The speaker should try to conclude with a discussion of the structure theorem for perfectoid rings.

2.5. **The prismatic site.** The speaker will define the prismatic site, and define the basic structure sheaves on this site. From here, we can easily define the so-called prismatic complex and the Hodge-Tate complex in separate derived categories, whose cohomologies we will call the prismatic/Hodge-Tate cohomologies. We then proceed with a little more on the prismatic site, such as defining the left-adjoint to the forgetful functor from prisms over a prism (A, I) to δ-pairs over (A, I), called the *prismatic envelope*, and will likely reserve discussion of it for later when we get to the Hodge-Tate comparison theorems. We aim to conclude with a discussion of products in the prismatic site, and a few more remarks.

2.6. **The Hodge-Tate and crystalline comparison theorems.**

2.7. **Derived prismatic cohomology.**

2.8. **Perfections in mixed characteristic.**

2.9. **The étale comparsion theorem.**

2.10. **The q-de Rham complex.**

2.11. **q-crystalline cohomology.**

2.12. **Prismatic cohomology via THH.**

References

1. Bhargav Bhatt, *Eilenberg lectures on prismatic cohomology*, (2018).
2. Bhargav Bhatt and Peter Scholze, *Prisms and prismatic cohomology*, Ann. of Math. (2) **196** (2022), no. 3, 1135–1275. MR 4502597
3. Kiran Kedlaya, *Notes on prismatic cohomology*, (2021).